

SOLUCIONES DE MATRICES CON INCÓGNITAS

1. Calcular el valor de X en las siguientes ecuaciones:

$$A = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

a) $XA = B + I$

$$X \cdot A \cdot A^{-1} = (B+I) \cdot A^{-1}$$

$$X \cdot I = (B+I) \cdot A^{-1}$$

$$X = (B+I) \cdot A^{-1}$$

$$B + I = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{\left(\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}^* \right)^t}{\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}} = \frac{\begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix}^t}{1} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 9 & -2 \\ -2 & 1 \end{pmatrix}$$

b) $AX + B = C$

$$AX = C - B$$

$$A^{-1} \cdot A \cdot X = A^{-1} \cdot (C - B)$$

$$I \cdot X = A^{-1} \cdot (C - B)$$

$$X = A^{-1} \cdot (C - B)$$

Recordamos, del apartado anterior que $A^{-1} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$

$$C - B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\mathbf{c) \quad XAB - XC = 2C}$$

$$X(AB-C) = 2C$$

$$X \cdot (AB-C) \cdot (AB-C)^{-1} = 2C(AB-C)^{-1}$$

$$X \cdot I = 2C(AB-C)^{-1}$$

$$X = 2C(AB-C)^{-1}$$

$$2C = \begin{pmatrix} 2 & 4 \\ 2 & 6 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 10 & 8 \end{pmatrix}$$

$$A \cdot B - C = \begin{pmatrix} 3 & 2 \\ 10 & 8 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 9 & 5 \end{pmatrix}$$

$$(A \cdot B - C)^{-1} = \frac{\left(\begin{pmatrix} 2 & 0 \\ 9 & 5 \end{pmatrix}^* \right)^t}{\begin{vmatrix} 2 & 0 \\ 9 & 5 \end{vmatrix}} = \frac{\begin{pmatrix} 5 & -9 \\ 0 & 2 \end{pmatrix}^t}{10} = \frac{\begin{pmatrix} 5 & 0 \\ -9 & 2 \end{pmatrix}}{10} = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{9}{10} & \frac{1}{5} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 2 & 6 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{9}{10} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} -26 & 4 \\ 10 & 6 \\ 44 & 6 \\ -10 & 5 \end{pmatrix}$$